CAT 2023 – Slot 3 Paper (Memory Based)

Section: Quantitative Aptitude

Q.1) If x is a positive real number such that $x^8 + \left(\frac{1}{x}\right)^8 = 47$, then the value of $x^9 + \left(\frac{1}{x}\right)^9$ is

A. $34\sqrt{5}$

B. $40\sqrt{5}$

C. $36\sqrt{5}$

D. $30\sqrt{5}$

Solution:

Best suggestion would be to skip such questions in the exam, but here is the solution

using $a^2 + b^2 = (a + b)^2 - 2ab$ sequentially

$$x^8 + \left(\frac{1}{x}\right)^8 = \left(x^4 + \left(\frac{1}{x}\right)^4\right)^2 - 2 = \left(\left(x^2 + \left(\frac{1}{x}\right)^2\right)^2 - 2\right)^2 - 2$$

$$\Rightarrow x^8 + \left(\frac{1}{x}\right)^8 = \left[\left(\left(x + \frac{1}{x}\right)^2 - 2\right)^2 - 2\right]^2 - 2$$

let's call $\left(x + \frac{1}{x}\right) = t$

$$\Rightarrow x^8 + \left(\frac{1}{x}\right)^8 = \left[\left(t^2 - 2\right)^2 - 2\right]^2 - 2 = 47$$

$$\Rightarrow \left[\left(t^2 - 2\right)^2 - 2\right]^2 = 49 \Rightarrow \left(t^2 - 2\right)^2 = 9 \Rightarrow t^2 = 5$$

$\Rightarrow\ t = x + \frac{1}{x} = \sqrt{5}$ (can't be negative as it can be always expressed as $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 + 2$)

Now let's use $a^3 + b^3 = (a+b)^3 - 3a^2b - 3ab^2$ sequentially

$$x^9 + \left(\frac{1}{x}\right)^9 = \left(x^3 + \frac{1}{x^3}\right)^3 - 3\left(x^3 + \frac{1}{x^3}\right)$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} = (\sqrt{5})^3 - 3\sqrt{5} = 2\sqrt{5}$$

$$x^9 + \left(\frac{1}{x}\right)^9 = (2\sqrt{5}\,)^3 - 3(2\sqrt{5}) = 34\sqrt{5}$$

Q.2) Let n and m be two positive integers such that there are exactly 41 integers greater than 8^m and less than 8^n, which can be expressed as powers of 2. Then, the smallest possible value of n+m is

A. 44
B.16
C. 42
D. 14

Solution:

$8^m = 2^{3m};\ 8^n = 2^{3n}$

Now next integer to 8^m which can be expressed as power of 2 would be 2^{3m+1}. Similar way series would be

$2^{3m},\ 2^{3m+1}, 2^{3m+2}, 2^{3m+3}$,............, 2^{3n}

$$3n = 3m + 42$$

$$n - m = 14$$

$$n + m = 2m + 14$$

Minimum value of m = 1 (positive integer)

Q.3) For some real numbers a and b, the system of equation $x + y = 4$ and $(a + 5)x + (b^2 - 15)y = 8b$ has infinitely many solutions for x and y. then, the maximum possible value of ab is

A. 15
B. 55
C. 33
D. 25

Solution:
For infinitely many solutions

$$\frac{1}{a+5} = \frac{1}{b^2-15} = \frac{4}{8b}$$

$b^2 - 15 = 2b \Rightarrow b = 5\ or\ -3$
$a = 5\ or\ -11$

Maximum possible value of ab = 33

Q.4) For a real number x , If $\frac{1}{2}$, $\frac{log_3(2^x-9)}{log_3 4}$, and $\frac{log_5(2^x+\frac{17}{2})}{log_5 4}$ are in an arithmetic progression, then the common difference is

A. $log_4\left(\frac{23}{2}\right)$

B. $log_4\left(\frac{3}{2}\right)$

C. $log_4 7$

D. $log_4\left(\frac{7}{2}\right)$

Solution:

$$2 * \frac{log_3(2^x-9)}{log_3 4} = \frac{1}{2} + \frac{log_5(2^x+\frac{17}{2})}{log_5 4}$$

Let's try to make bases for log same

$$2 * \frac{log(2^x-9) * log3}{log3 * log4} = \frac{1}{2} + \frac{log(2^x+\frac{17}{2}) * log5}{log5 * log4} \Rightarrow 2 * \frac{log(2^x-9)}{log4} = \frac{1}{2} + \frac{log(2^x+\frac{17}{2})}{log4}$$

$\Rightarrow\ 2 * log(2^x - 9) = log2 + log(2^x + \frac{17}{2})$

$\Rightarrow\ (2^x - 9)^2 = 2 * (2^x + \frac{17}{2})$

Use $t = 2^x$

$(t - 9)^2 = 2t + 17 \quad \Rightarrow t^2 - 20t + 64$

$t = 4\ or\ 16$ but 4 is rejected as $2^x - 9$ can't be negative
So $x = 4$

Now common difference = $\frac{log_3(2^x-9)}{log_3 4} - \frac{1}{2} = log_4(7) - 1/2 = log_4(\frac{7}{2})$

Q.5) A quadratic equation $x^2 + bx + c = 0$ has two real roots. If the difference between the reciprocals of the roots is 1/3, and the sum of the reciprocals of the square of the roots is 5/9, then the largest possible value of (b+c) is

Solution:

$\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{5}{9} \Rightarrow \frac{(\beta+3)^2}{(3\beta)^2} + \frac{1}{\beta^2} = \frac{5}{9} \Rightarrow \frac{\beta^2 + 9 + 6\beta + 9}{9\beta^2} = \frac{5}{9}$

$\Rightarrow 2\beta^2 - 3\beta - 9 = 0 \Rightarrow (\alpha, \beta) = (3/2, 3)\ or\ (-3, -3/2)$

$b + c = -(\alpha + \beta) + \alpha\beta$

maximum value of b+c = 9

Q.6) The sum of the first two natural numbers, each having 15 factors (including 1 and the number itself) is

Solution:

Number could be represented as $N = a^p * b^q * c^r \ldots$ where a,b and c are prime numbers and p,q,r,.. are integers > 0

For every factor you can take either with power 0, 1, 2,...... maximum p. Similar with b, c and other prime factors. Every combination will give you a factor of N.

Now total number of factors for N are (p+1)*(q+1)*(r+1)*.... if we include 1 and N as well.

Now given (p+1)*(q+1)*(r+1)*..... = 15 = 3*5

Only possibility is p = 2 and q = 4. r and other terms will not exist, so we have $N = a^2 * b^4$

N would be smallest if a = 3 and b = 2; $N = 3^2 * 2^4 = 144$ (we have plugged in smallest prime number 2 with higher power, and second smallest prime number 3 with lower power)

Second smallest number would be if a=5 and b=2 (since we have already used 3 in the smallest number, the next option is plugging 5 next prime number after 3). $N = 5^2 * 2^4 = 400$

Another option could also be to try a = 2 and b = 3. $N = 2^2 * 3^4 = 324$; which is smaller than 400

So our final numbers are 144 and 324.

Q.7) Let n be any natural number such that $5^{n-1} < 3^{n+1}$. Then, the least integer value of m that satisfies $3^{n+1} < 2^{n+m}$ for each such n is

Solution:

With every increase in 'n' 5^{n-1} will increase much faster than 3^{n+1} (since you are multiplying left term with 5 but right term with only 3 as you increase n). At $n = 1,\ 5^0 < 3^2$; but as n increases 5^{n-1} will cross 3^{n+1} at some value of n. Trying values of n, we see that till n=5, inequality holds and then from n=6, $5^{n-1} > 3^{n+1}$

Now again with increase in n, 3^{n+1} increases at a much faster rate than 2^{n+m}, so let;s try the biggest value of n which is 5.
$3^{5+1} = 3^6$ and 2^{5+m}, Now for 2^{5+m} to be greater than 3^6 = 729; 5+m must be 10 (
$2^9 = 512\ and\ 2^{10} = 1024$). So m must be 5

Q.8) A boat takes 2 hours to travel downstream a river from port A to port B, and 3 hours to return to port A. Another boat takes a total of 6 hours to travel from port B to port A and return to port B. If the speeds of the boats and the river are constant, then the time, in hours, taken by the slower boat to travel from port A to port B is

A. $3\left(\sqrt{5}-1\right)$

B. $3\left(3+\sqrt{5}\right)$

C. $3\left(3-\sqrt{5}\right)$

D. $12\left(\sqrt{5}-2\right)$

Solution:

Let's say speeds are B_1, B_2 and R . Also let's say time taken by slower boat to travel from A to B is 't'

Now,

$(B_1 + R) * 2 = (B_1 - R) * 3$

$B_1 = 5R$

$(B_1 + R) * 2 = (B_2 + R) * t = (B_2 - R)(6 - t)$

$12R = (B_2 + R) * t = (B_2 - R)(6 - t)$

$12R = (B_2 - R)(6 - t) \Rightarrow t = 6 - \frac{12R}{B_2 - R}$

Now

$12R = \left[B_2 + R\right] * \left[6 - \frac{12R}{B_2 - R}\right]$

$2R * B_2 - 2R^2 = B_2^2 - 2B_2 * R - 3R^2$

$B_2^2 - 4R * B2 - R^2 = 0$

$B_2 = \frac{4+\sqrt{20}}{2}R = (2 + \sqrt{5})R$

Now, using these values above

$t = 6 - \frac{12R}{(2+\sqrt{5})R - R} = 6 - \frac{12}{1+\sqrt{5}} = 6 - \frac{12(1-\sqrt{5})}{-4} = 9 - 3\sqrt{5}$

Q.9) Anil mixes cocoa with sugar in the ratio 3:2 to prepare mixture A, and coffee with sugar in the ratio 7:3 to prepare mixture B. He combines mixtures A and B in the ratio 2:3 to make a new mixture C. If he mixes C with an equal amount of milk to make a drink, then the percentage of sugar in this drink will be

A. 16
B. 24
C. 17
D. 21

Solution: Let;s say he mixes 2x and 3x amounts of A & B to make 5x of C.

So in C, we have sugar = $\frac{2}{5} * 2x + \frac{3}{10} * 3x = \frac{17}{10}x$ out of 5x mixture which is being mixed with 5x milk

So percentage of sugar = $\frac{\frac{17}{10}x}{10x} * 100 = 17\%$

Q.10) Rahul, Rakshita and Gurmeet, working together, would have taken more than 7 days to finish a job. On the other hand, Rahul and Gurmeet, working together, would have taken less than 15 days to finish the job. However, they all worked together for 6 days, followed by Rakshita, who worked alone for 3 more days to finish the job. If Rakshita had worked alone on the job then the number of days she would have taken to finish the job, cannot be

A. 16
B. 21
C. 17
D. 20

Solution: Say their work rate of Rahul, Rakshita, Gurmeet is a, b and c

$(a + b + c) * 7 < 1$

$(a + c) * 15 > 1$

$(a + b + c) * 6 + b * 3 = 1 \Rightarrow (a + c) = \frac{1}{6} - \frac{3}{2}b$

Using this in above equations

$\left(\frac{1}{6} - \frac{3}{2}b\right) * 15 > 1 \quad \Rightarrow b < \frac{1}{15}$

$(\frac{1}{6} - \frac{3}{2}b + b) * 7 < 1 \Rightarrow \frac{1}{21} < b$

So Rakshita can complete the work somewhere between 15 to 21 days. Answer is B

Q. 11) The population of a town in 2020 was 100000 . The population decreased by % y from the year 2020 to 2021, and increased by % x from the year 2021 to 2022, where x and y are two natural numbers. If population in 2022 was greater than the population in 2020 and the difference between x and y is 10 , then the lowest possible population of the town in 2021 was
A. 74000
B. 75000
C. 73000
D. 72000

Solution:

$(1-\frac{x}{100})*(1+\frac{y}{100})>1$ (Since the population has increased)

Also y > x, as population decreased and then increased to a higher number

Q. 12) There are three people , A, B and C in a room. If a person D joins the room, the average weight of the persons in the room reduces by x. Instead of D, if person E joins the room, the average weight of the persons in the room increases by 2x . If the weight of E is 12 kg more than that of D, then the value of x is
A. 1.5
B. 0.5
C. 1
D. 2

Solution:

$e = d + 12$

$\frac{a+b+c+d}{4}-\frac{a+b+c}{3}=-x \Rightarrow \frac{a+b+c}{12}-\frac{d}{4}=x \Rightarrow \frac{a+b+c}{12}-\frac{e}{4}+3=x$

$\frac{a+b+c+e}{4}-\frac{a+b+c}{3}=2x \Rightarrow \frac{e}{4}-\frac{a+b+c}{12}=2x$

Add both the equations to get 3x = 3 or x = 1

Q. 13) A merchant purchases a cloth at a rate of Rs. 100 per meter and receives 5 cm length of cloth free for every 100 cm length of cloth purchased by him. He sells the same cloth at a rate of Rs.110 per meter but cheats his customers by giving 95 cm length of cloth for every 100 cm length of cloth purchased by the customers. If the merchant provides a 5% discount, the resulting profit earned by him is
A. 9.7%
B. 16%
C. 4.2%
D. 15.5%

Solution:

cost price of 100m of the cloth = $100/(100 + 5) = 20/21$

Selling price of 100m of the cloth = $110/(100 - 5) * 0.95 = 11/10$

Profit = $\frac{(11/10)-(20/21)}{20/21} * 100 = \frac{231-200}{200} * 100 = 15.5\%$

Q. 14) Gautam and Suhani, working together, can finish a job in 20 days. If Gautam does only 60% of his usual work on a day, Suhani must do 150% of her usual work on that day to exactly make up for it. Then, the number of days required by the faster worker to complete the job working alone is

Solution:

Let's say their rate of work is g and s. Since 40% less work for Gautam is equivalent to 50% work done by Suhani

0.4*g = 0.5*s
g = (5/4) s; Gautam is faster than Suhani

(g+s)*20 = 1
Use above relation to deduce that s = 1/45 and g = 1/36

Gautam will take 36 days and Suhani will take 45 days to finish the job doing all alone

Q. 15) The number of coins collected per week by two coin-collectors A and B are in the ratio 3 : 4. If the total number of coins collected by A in 5 weeks is a multiple of 7 , and the total number of coins collected by B in 3 weeks is a multiple of 24 , then the minimum possible number of coins collected by A in one week is

Solution:

let's say A collected '7m' coins in 5 weeks and B collected '24n' coins in 3 weeks

$\left(\frac{7m}{5}\right):\left(\frac{24n}{3}\right) = 3:4 \Rightarrow m = \frac{30}{7}n$

For m and n to be integers, minimum values m = 30 and n = 7

So A collected 30*7/5 = 42 coins in one week

Q. 16) A fruit seller has a stock of mangoes, bananas and apples with at least one fruit of each type. At the beginning of a day, the number of mangoes make up 40% of his stock. That day, he sells half of the mangoes, 96 bananas and 40% of the apples. At the end of the day, he ends up selling 50% of the fruits. The smallest possible total number of fruits in the stock at the beginning of the day is

Solution:
Initial stock let it be m, b and a

m = 0.4*(m+b+a) = 0.4*T ; where T is total
Note that T must be a multiple of 5 to make m an integer

Also,
0.5*m + 96 + 0.4*a = 0.5*T

0.2T + 96 + 0.4a = 0.5T

This means 0.5*m must be an integer which means m must be a multiple of 2.
0.5T must be an integer that means T is also a multiple of 2.
0.4a must be an integer so a must be a multiple of 5.

T = 320 + (4/3)a
a must be a multiple of 3 as well to satisfy the equation

Lets try a = 3*5 = 15
T = 320 + 20 = 340

m = 136
b = 189

Everything seems to satisfy the conditions

Q.17) Let ABC be an isosceles triangle such that AB and AC are of equal length. AD is the altitude from A on BC and BE is the altitude from B on AC. If AD and BE intersect at O such that $\angle AOB = 105^0$, then $\frac{AD}{BE}$ equals

A. $\sin 15^0$

B. $\cos 15^0$

C. $2\cos 15^0$

D. $2\sin 15^0$

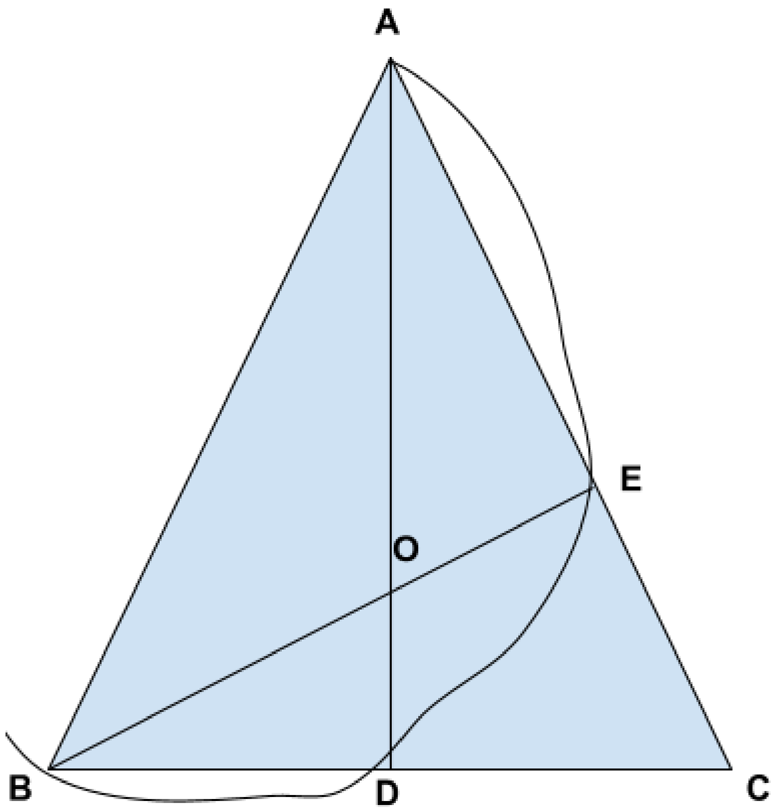
Solution:

$\angle B = \angle C$

Since ∠AEB and ∠ADB are right angles, we can imagine a circle passing through E & D with AB as diameter.

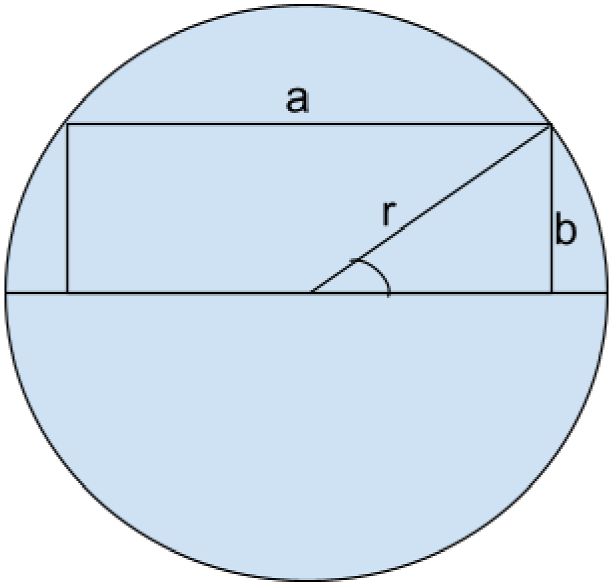
∠AOB = 105^0 = ∠DOE (opposite angles)
∠C = 360 - (90 + 90 + 105) = 75^0 = ∠B
∠A = 180 - 2*75 = 30^0
∠BAD = 15^0

$\frac{AD}{BE} = \frac{AD}{AB} * \frac{AB}{BE} = cos(15) * \frac{1}{sin(30)} = 2cos(30)$ (remember sin30 = ½)



Q.18) A rectangle with the largest possible area is drawn inside a semicircle of radius 2 cm. Then, the ratio of the lengths of the largest to the smallest side of this rectangle is
A. 1:1
B. $\sqrt{5}:1$
C. $\sqrt{2}:1$
D. 2:1

Solution:



$(a/2)^2 + (b)^2 = r^2$

Area = $\frac{1}{2} * ab = r * cos(\theta) * r * sin(\theta) = r^2 sin(2\theta)/2$ (using basic trigonometry)

Area is maximum when $\theta = 45^0$

$\Rightarrow \frac{a}{2} = b$

Solving it we get

$b = r/\sqrt{2}$; and $a = \sqrt{2}r$

Q. 19) In a regular polygon, any interior angle exceeds the exterior angle by 120 degrees. Then, the number of diagonals of this polygon is

Solution:

For a regular polygon interior angle = $180^0 - \frac{360^0}{n}$

And exterior angle = $\frac{360^0}{n}$

$180^0 - \frac{360^0}{n} - \frac{360^0}{n} = 120^0$

n = 12

And diagonals of a regular polygon = $nC2 - n = 54$

Q.20 Let $a_n = 46 + 8n$, and $b_n = 98 + 4n$ be two sequences for natural numbers $n \leq 100$. Then, the sum of all terms common to both the sequences is
A. 14900
B. 15000
C. 14798
D. 14602

Solution:

Let's write second series: 98, 102, 106, 110, 114
102 satisfies the first series which will be the first term for the common series

Now since the first series moves at a common difference of 8 and second one at 4, the common series will move at the common difference of LCM of both which is 8.

So common series becomes

$c_n = 102 + 8(n - 1)$

We also need to see the biggest term of this series. So let's go back to the biggest term of series b which is = 98 + 400 = 498. Note that the biggest term of series a is going to be bigger than that of b because it has double the common difference and not very big difference in the first term.

Now let;s write series b from biggest terms in decreasing order: 498, 494, 490, 486

494 satisfies the other series which is biggest term of our common series as well

494 = 102 + 8(n-1)
n = 50

Use AP sum formula to get $= \frac{50}{2} * (2 * 102 + 49 * 8) = 14900$

Q.21) The value of $1 + \left(1 + \frac{1}{3}\right)\frac{1}{4} + \left(1 + \frac{1}{3} + \frac{1}{9}\right)\frac{1}{16} + \left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}\right)\frac{1}{64} +$...... is
A. 15/13
B. 27/12
C. 16/11
D. 15/8

Solution: Standard AGP

$S = 1 + \left(1 + \frac{1}{3}\right)\frac{1}{4} + \left(1 + \frac{1}{3} + \frac{1}{9}\right)\frac{1}{16} + \left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}\right)\frac{1}{64} +$.....

$$S.\left(\frac{1}{4}\right) = \frac{1}{4} + \left(1+\frac{1}{3}\right)\frac{1}{16} + \left(1+\frac{1}{3}+\frac{1}{9}\right)\frac{1}{64} + \left(1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}\right)\frac{1}{64*4} + ...$$

Subtract Eq. 2 from Eq. 1

$$S\left(1-\frac{1}{4}\right) = 1 + \frac{1}{3}*\frac{1}{4} + \frac{1}{9}*\frac{1}{16} + \frac{1}{27}*\frac{1}{64} + ...$$

$$S\left(1-\frac{1}{4}\right) = 1 + \frac{1}{3}*\frac{1}{4} + \frac{1}{9}*\frac{1}{16} + \frac{1}{27}*\frac{1}{64} + ...$$

$$S\left(\frac{3}{4}\right) = 1 + \frac{1}{3*4} + \frac{1}{(3*4)^2} + ... = \frac{1}{1-\frac{1}{12}} = \frac{12}{11}.$$

$$S = 16/11$$

Q. 22) Suppose f(x,y) is a real-valued function such that f(3x+2y,2x-5y)=19x , for all real numbers x and y. The value of x for which f(x,2x)=27 , is

Solution:

Since $f(3x + 2y,\ 2x - 5y)$ is independent of y. So, let's try to arrive at a function that eliminates y in this case. While there could be thousand of ways, let;s try with linear polynomials

$$f(3x + 2y,\ 2x - 5y) = 5 * (3x + 2y) + 2 * (2x - 5y) = 19x$$

Now $f(x,\ 2x) = 5x + 2 * 2x = 9x = 27$

$\Rightarrow x = 3$

Answer Keys

Q. No.	QUANT
1	A
2	B
3	C
4	D
5	9
6	468
7	5
8	C
9	C
10	B
11	C
12	C
13	D
14	36
15	42
16	340
17	C
18	D
19	54
20	A
21	C
22	3