CAT 2023 – Slot 2 Paper (Memory Based)

Section 03: Quantitative Aptitude

Q.1) For any natural numbers m, n, and k such that k divides both m+2n and 3m+4n, k must be a common divisor of

A. m and n

B. 2m and 3n

C. 2m and n

D. m and 2n

Solution:

3m + 4n = m + 2*(m+2n)

If it divides (3m+4n) and (m+2n), then it must divide m as well from above equation

Now, if it divides m+2n and m then it must divide 2n as well

Q.2) Any non-zero real numbers x, y such that $y \neq 3$ and $\frac{x}{y} < \frac{x+3}{y-3}$, will satisfy the condition

A. $\frac{x}{y} < \frac{y}{x}$

B. If y > 10 , then -x > y

C. If x < 0, then -x < y

D. If y < 0, then -x < y

Solution:

$\frac{x+3}{y-3} - \frac{x}{y} > 0 \Rightarrow \frac{y(x+3)-x(y-3)}{y(y-3)} > 0 \Rightarrow \frac{(y+x)}{y(y-3)} > 0$

Take cases

y>3 denominator is positive, y > -x

y is between 0 and 3, denominator is negative so y < -x

y < 0 means denominator is positive, so y > -x

Q.3) The sum of all possible values of x satisfying the equation $2^{4x^2} - 2^{2x^2+x+16} + 2^{2x+30} = 0$, is

A. 5/2

B. 1/2
C. 3
D. 3/2

Solution:

$2^{4x^2} - 2^{2x^2+x+16} + 2^{2x+30} = 0$

$(2^{2x^2})^2 - 2 * 2^{2x^2} * 2^{x+15} + (2^{x+15})^2 = 0$

$(2^{2x^2} - 2^{x+15}) = 0$

$2^{2x^2} = 2^{x+15}$

$2x^2 = x + 15$

$2x^2 - 6x + 5x - 15 = 0$

$(2x + 5)(x - 3) = 0$

Q.4) Let a, b m and n be the natural numbers such that a>1 and b>1. If $a^m b^n = 144^{145}$, then the largest possible value of n-m is

A. 579

B. 289

C. 580

D. 290

Solution:

$144^{145} = 2^{580} * 3^{290} = 2^{580} * (3^{290})^1$

maximum value of n-m = 580 - 1 = 579

Q.5) The number of positive integers less than 50, having exactly two distinct factors other than 1 and itself, is

Solution:

N = m*n, so m & n needs to be prime to make it exactly two distinct factors

Since 7*7 = 49, we can pick any two prime numbers till 7 (2,3,5,7) to multiply = $4C2$ = 6

Next prime number is 11, so 11*2 and 11*3

With 13, 13*2, 13*3

With 17, 17*2

With 19, 19*2

With 23, 23*2

And we are done counting = 13

There could also be cases where a number is expressed as a cube of a prime number so m and m^2 are distinct like (2, 4). Only other such cases could be (3, 9).

So total = 13 + 2 = 15

Q.6) Let k be the largest integer such that the equation $(x - 1)^2 + 2kx + 11 = 0$ has no real roots. If y is a positive real number, then the least possible value of $\frac{k}{4y} + 9y$ is

Solution:

$x^2 + x(2k - 2) + 12 = 0$

$D = (2k - 2)^2 - 4 * 12 = 4k^2 - 8k - 44 < 0$

$k^2 - 2k - 11 < 0$

$(k - 1)^2 < 12$

Largest integer value of 'k' satisfying this is 4

Now substituting k = 4, we have to minimize $\frac{k}{4y} + 9y = \frac{1}{y} + 9y = \left(\frac{1}{\sqrt{y}} - 3\sqrt{y}\right)^2 + 6$

So minimum value of the expression can be 6 when $\left(\frac{1}{\sqrt{y}} - 3\sqrt{y}\right)^2$ becomes 0

Q.7) For some positive real number x, if $log_{\sqrt{3}}(x) + \frac{log_x 25}{log_x(0.008)} = \frac{16}{3}$, then the value of $log_3(3x^2)$ is

Solution:

Simplify all the expressions

$2log_3 x + \frac{2log_x 5}{-3log_x 5} = \frac{16}{3}$

$2log_3 x = 6$

$x^2 = 3^6$

$log_3(3x^2) = log_3(3 * 3^6) = 7$

Q.8) Pipes A and C are fill pipes while Pipe B is a drain pipe of a tank. Pipe B empties the full tank in one hour less than the time taken by Pipe A to fill the empty tank. When pipe A, B and C are turned on together, the tank is filled in two hours. If pipes B and C are turned on together when the tank is empty and pipe B is turned off after one hour, then pipe C takes another one hour and 15 minutes to fill the remaining tank. If the pipe A can fill the empty tank in less than five hours, then the time taken, by pipe C to fill the empty tank is

A. 75

B. 120

C. 60
D. 90

Solution:

Let;s say time taken by the pipes to completely fill/ empty the tank is a, b and c

Eq (1) $b + 1 = a$

Eq (2) $\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c}\right) * 2 = 1$

Eq (3) $\left(\frac{1}{c} - \frac{1}{b}\right) + \frac{1}{c} * \frac{5}{4} = 1 \Rightarrow \frac{9}{4c} = 1 + \frac{1}{b} \Rightarrow c = \frac{9b}{4(1+b)}$

Eq (4) $a < 5$

Use 1 & 3 in 2:-

$\left(\frac{1}{b+1} - \frac{1}{b} + \frac{4(1+b)}{9b}\right) * 2 = 1$

$\frac{9(b)-9(b+1)+4(1+b)^2}{9b(b+1)} = \frac{1}{2}$

$\frac{4b^2+8b-5}{9b^2+9b} = \frac{1}{2}$

$8b^2 + 16b - 10 = 9b^2 + 9b$

$b^2 - 7b + 10 = 0$

So b = 2 or 5 but a <5, that means b = 2

and c = $\frac{9*2}{4*(1+2)}$ = 1.5 hr = 90 mins

Q.9) Ravi is driving at a speed of 40 Km/h on a road. Vijay is 54 meters behind Ravi and driving in the same direction as Ravi. Ashok is driving along the same road from the opposite direction at a speed of 50 Km/h and 225 meters away from Ravi. The speed, in Km/h, at which Vijay should drive so that all the three cross each other at the same time, is

A. 67.2

B. 64.4

C. 61.6

D. 58.8

Solution:

Time taken by Ravi & Ashok to meet = $\frac{0.225}{40+50}$ (use relative speed concept to cover the distance & don't forget to convert distance into km)

Now in the same time Vijay & Ravi should also meet, so if Vijay's speed is 's', then he would need to be faster than Ravi to cover the extra 54m distance

$(s - 40) * \frac{0.225}{90} = 0.054$

$s - 40 = \frac{54*90}{225} = 21.6$

So Vijay's speed is 61.6

Q.10) In a company, 20% of the employees work in the manufacturing department. If the total salary obtained by all the manufacturing employees is one-sixth of the total salary obtained by all the employees in the company, then the ratio of the average salary obtained by the manufacturing employees to the average salary obtained by the non-manufacturing employees is

A. 6 : 5

B. 4 : 5

C. 5 : 4

D. 5 : 6

Solution: Let;s say average salaries are 'm' & 'n' for manufacturing and non manufacturing folks

$0.2m = \frac{1}{6}(0.2m + 0.8n)$

Q.11) Minu purchases a pair of sunglasses at Rs.1000 and sells to Kanu at 20% profit. Then, Kanu sells it back to Minu at 20% loss. Finally, Minu sells the same pair of sunglasses to Tanu. If the total profit made by Minu from all her transactions is Rs.500, then the percentage of profit made by Minu when she sold the pair of sunglasses to Tanu is

A. 26%

B. 35.42%

C. 52%

D. 31.25%

Solution:

Minu sold it to Kanu at = 1000*1.2 = 1200

Kanu sold it to Minu at =1200*0.8 = 960

Minu sold it to Tanu at = x

$(x - 960) + (1200 - 1000) = 500$

$x = 1260$

Profit% = $\frac{1260-960}{960} * 100 = 31.25\%$

Q. 12) The price of a precious stone is directly proportional to the square of its weight. Sita has a precious stone weighing 18 units. If she breaks it into four pieces with each piece having distinct integer weight, then the difference between the highest and lowest possible values of the total price of the four pieces will be 288000. Then, the price of the original precious stone is

A. 1620000

B. 1296000

C. 1944000

D. 972000

Solution:

$p = kw^2$; *where p is the price, w is the weight and k is a constant*

Now if broken into four pieces, total price would be

$p_{total} = p_1 + p_2 + p_3 + p_4 = k\left(w_1^{\ 2} + w_2^{\ 2} + w_3^{\ 2} + w_4^{\ 2}\right)$

Maximum value of price would be realized if there is one piece which is the biggest possible and rest being small, and minimum value would be when units are closer to each other (since equal is not an option). This is from your understanding of squares of integers.

Maximum case: 1,2,3,12

Minimum case: 3,4,5,6

$p_{max} = k(1^2 + 2^2 + 3^2 + 12^2) = 158k$

$p_{min} = k(3^2 + 4^2 + 5^2 + 6^2) = 86k$

$p_{max} = p_{min} = 72k = 288000 \Rightarrow k = 4000$

$p_{original} = 4000 * (18)^2 = 1296000$

Q. 13) Anil borrows Rs 2 lakhs at an interest rate of 8% per annum, compounded half-yearly. He repays Rs 10320 at the end of the first year and closes the loan by paying the outstanding amount at the end of the third year. Then, the total interest, in rupees, paid over the three years is nearest to

A. 33130

B. 40991

C. 51311

D. 45311

Solution:

Loan outstanding at the end of the 1 year = $200000 * \left(1 + \frac{4}{100}\right)^2 - 10320 = 206000$

Loan outstanding at the end of the 3 year = $206000 * \left(1 + \frac{4}{100}\right)^4 = 240991$

Total interest = $240991 + 10320 - 200000 = 51311$

Q.14) A container has 40 liters of milk. Then, 4 liters are removed from the container and replaced with 4 liters of water. This process of replacing 4 liters of the liquid in the container with an equal volume of water is continued repeatedly. The smallest number of times of doing this process, after which the volume of milk in the container becomes less than that of water, is

Solution:

Let;s see if we can create a pattern for nth term

Volume of Milk after first replacement: $= 40 - 4 = 36$

Volume of Milk after 2nd replacement: $= (40 - 4) - 4 * \left(\frac{40-4}{40}\right) = (40 - 4)\left(1 - \frac{4}{40}\right)$

Volume of Milk after 3rd replacement: $= \left[(40 - 4)\left(1 - \frac{4}{40}\right)\right] - 4 * \frac{\left[(40-4)\left(1-\frac{4}{40}\right)\right]}{40}$

$= 40 * \left(1 - \frac{4}{40}\right)^3$

Volume of Milk after nth replacement $V_{milk}(n) = 40 * \left(\frac{9}{10}\right)^n$

Since it’s only milk and water, if milk goes less than 50% water will be more than milk. So let’s solve for volume = 20 L

$40 * \left(\frac{9}{10}\right)^n \leq 20$

$\left(\frac{9}{10}\right)^n \leq \frac{1}{2}$

Check by trial and error to arrive at the answer

Q. 15) Jayant bought a certain number of white shirts at the rate of Rs 1000 per piece and a certain number of blue shirts at the rate of Rs 1125 per piece. For each shirt, he then set a fixed market price which was 25% higher than the average cost of all the shirts. He sold all the shirts at a discount of 10% and made a total profit of Rs 51000. If he bought both colors of shirts, then the maximum possible total number of shirts that he could have bought is

Solution:

Let's say he bought 'w' white shirts and 'b' blue shirts

average cost = $\frac{1000w + 1125b}{w+b}$

market price = $\frac{1000w + 1125b}{w+b} * 1.25$

SP = $\frac{1000w + 1125b}{w+b} * 1.25 * 0.9 = \frac{9}{8} * \frac{1000w + 1125b}{w+b}$

Profit = $\frac{9}{8} * \frac{1000w + 1125b}{w+b} * (w + b) - (1000w + 1125b) = 51000$

$125w + \frac{1125}{8}b = 51000$

This is equation of a line in first quadrant, the only difference is that values are only integers

Maximize $b + w = b + 408 - \frac{9}{8}b = 408 - \frac{b}{8}$

Since b can't be 0, b+w will be maximum for b = 8 to keep b and w also integers

b+w = 408-1 = 407

Q. 16) If a certain amount of money is divided equally among n persons, each one receives Rs 352 . However, if two persons receive Rs 506 each and the remaining amount is divided equally among the other persons, each of them receive less than or equal to Rs 330 . Then, the maximum possible value of n is

Solution:

Total money $352n \geq 2 * 506 + 330(n - 2)$

$352n - 330n \geq 2 * 506 - 660$

$22n \geq 352$

$n \geq 16$

Q. 17) In a rectangle ABCD, AB = 9 cm and BC = 6 cm. P and Q are two points on BC such that the areas of the figures ABP, APQ, and AQCD are in geometric progression. If the area of the figure AQCD is four times the area of triangle ABP, then BP : PQ : QC is

A. 1 : 1 : 2

B. 1 : 2 : 1

C. 1 : 2 : 4

D. 2 : 4 : 1

Solution:

Area of ABP = $\frac{1}{2} * (AB)(BP) = A$

Area of APQ = $\frac{1}{2} * (AB)(PQ) = Ar$

Area of AQCD = $\frac{1}{2} * (AB)(BQ) + (AB) * (QC) = (AB)\left(\frac{BP}{2} + \frac{PQ}{2} + QC\right) = Ar^2$

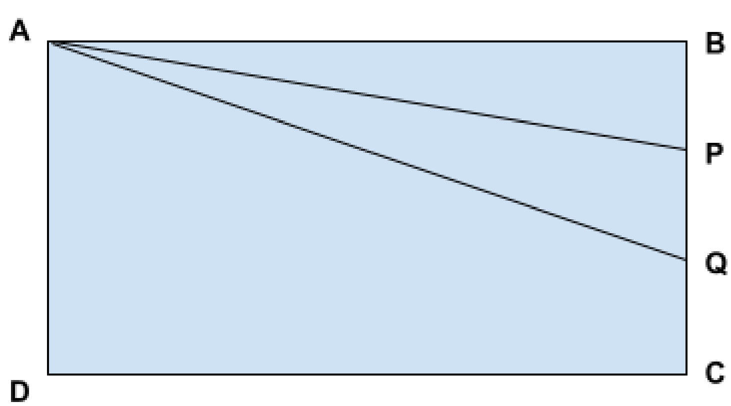
Area of AQCD = $Ar^2 = 4A \Rightarrow r = 2$

Substituting above

$A + Ar + (AB)(QC) = Ar^2 \Rightarrow A = (AB)(QC) = \frac{1}{2}(AB)(BP) = \frac{1}{4}(AB)(PQ)$

$4QC = 2BP = PQ$

$BP: PQ: QC = 2: 4: 1$



Q. 18) A triangle is drawn with its vertices on the circle C such that one of its sides is a diameter of C and the other two sides have their lengths in the ratio a:b. If the radius of the circle is r, then the area of the triangle is

A. $\frac{2abr^2}{a^2+b^2}$

B. $\frac{abr^2}{a^2+b^2}$

C. $\frac{abr^2}{2\left(a^2+b^2\right)}$

D. $\frac{4abr^2}{a^2+b^2}$

Solution:

If one of the sides is diameter that means it is a right angled triangle. Assume sides are ax and bx. Third side =2r (diameter)

area of the triangle would be $\frac{1}{2}(ax)(bx) = \frac{1}{2}abx^2$

$(ax)^2 + (bx)^2 = (2r)^2$ (great mathematician Mr. pythagoras to rescue)

Substituting we get Area = $\frac{ab(4r^2)}{2(a^2+b^2)} = \frac{2abr^2}{a^2+b^2}$

Q.19) The area of the quadrilateral bounded by the Y-axis, the line x=5, and the lines |x-y|-|x-5|=2, is

Solution:

Two given parallel lines are y axis which is x=0 and x=5

So we need to consider the range 0 to 5 for x for the inequality as that is where the lines will enclose the shape

$|x - y| - |x - 5| = 2$

Since $0 \leq x \leq 5$, it becomes

$|x - y| - 5 + x = 2$

$|x - y| + x = 7$

Now let's consider 2 cases to remove modulus

Case1: $x \geq y$

$x - y + x = 7$

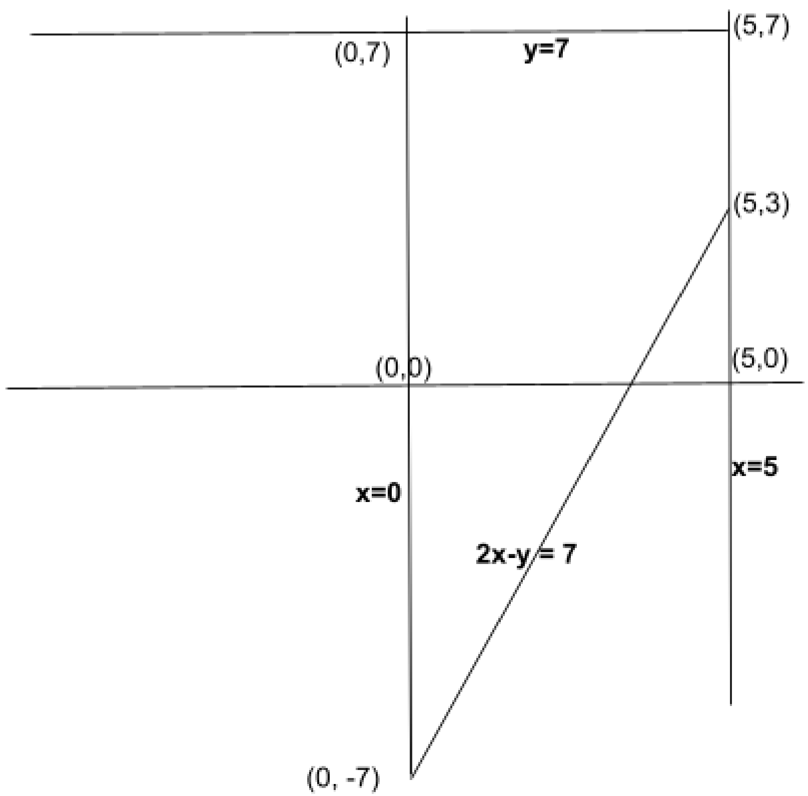
$2x - y = 7$ satisfies $x \geq y$ for $x \in [0, 5]$ $range$

Case2: $x \leq y$

$- x + y + x = 7$

$y = 7$ satisfies $x \leq y$ for $x \in [0, 5]$ $range$

Let's plot



It's a trapezium.

Area = $\frac{1}{2} * (4 + 14) * 5 = 45$

Q.20) Let both the series a_1, a_2, a_3,....... and b_1, b_2, b_3,....... are in arithmetic progression such that the common differences of both the series are prime numbers. If $a_5 = b_9$, $a_{19} = b_{19}$ and $b_2 = 0$, then a_{11} equals

A. 79

B. 83

C. 84

D. 86

Solution:

Let d and D be common differences for two series

$a + 4d = b + 8D$

$a + 18d = b + 18D$

$b + D = 0 \Rightarrow b =- D$

Substituting we get; $a + 4d = 7D$; and $a + 18d = 17D$

Solving it we get: $d = \frac{5}{7}D$

If common difference are prime numbers, then to satisfy above equation d = 5 and D = 7

b = -7 and a = 29

$a_{11} = a + 10d = 29 + 10 * 5 = 79$

Q.21) If $p^2 + q^2 - 29 = 2pq - 20 = 52 - 2pq$, then the difference between the maximum and minimum possible value of $(p - q)^3$ is

A. 486

B. 378

C. 243

D. 189

Solution:

$2pq - 20 = 52 - 2pq$

$4pq = 72 \Rightarrow pq = 18$

$p^2 + q^2 - 29 = 2pq - 20 \Rightarrow (p - q)^2 = 9 \Rightarrow p - q =\pm 3$

Also,

$p^2 + q^2 - 29 = 52 - 2pq \Rightarrow (p + q)^2 = 81 \Rightarrow p + q =\pm 9$

Solving this we get solution set of (p,q) as (6,3), (3,6), (-6,-3) and (-3,-6)

So maximum value of $(p - q)^3$ would be $(6 + 3)^3 - (- 6 - 3)^3 = 2 * 9^3 = 378$

Q.22) Let a_n and b_n be two sequences such that $a_n = 13 + 6(n - 1)$ and $b_n = 15 + 7(n - 1)$ for all numbers n. Then, the largest three-digit integer that is common to both these sequences, is

Solution:

Let's say that number occurs at nth term for a and mth term for b

$a_n = 13 + 6(n - 1) = 7 + 6n$

$b_m = 15 + 7(m - 1) = 8 + 7m$

First term is moving with a difference of 6 and second term is moving with a difference of 7, so if there are common terms then that sequence would move with a common difference of LCM of 6 and 7 which is 42

Let's call it $A_m = A_0 + 42n$

By trial and error we will need to find $A_0 = 43$

Now let's try $A_m = 999 = 43 + 42n \Rightarrow 42n = 956 \Rightarrow n = 22.76$....

Since 42 doesn't divide 956 completely, 999 can't be the term. But largest number that would satisfy would be when n is largest integer <22.76 which is 22

So term is 43 + 42*22 = 967

Another method could be to just try out all numbers reverse from for any sequence starting from biggest 3 digit and checking it if it satisfies the other sequence

Answer Keys

Q. No	Quant
1	D
2	D
3	B
4	A
5	15
6	6
7	7
8	D
9	C
10	B
11	D
12	B
13	C
14	7
15	407
16	16
17	D
18	A
19	45
20	A
21	B
22	967