CAT 2023 – Slot 1 Paper (Memory Based)

Quantitative Ability

Q.1) If x and y are real numbers such that $x^2 + (x - 2y - 1)^2 =- 4y(x + y)$, then the value $x - 2y$ is:

A. 1

B. 2

C. -1

D. 0

Solution:

$x^2 + x^2 + 4y^2 + 1 - 4xy - 2x + 4y = - 4yx - 4y^2$

$\Rightarrow 2x^2 + 8y^2 + 1 - 2x + 4y = 0$

$\Rightarrow 2x^2 + 8y^2 + 1 - 2x + 4y = 0$

$\Rightarrow (x + 2y)^2 + x^2 + 4y^2 + 1 - 4xy - 2x + 4y = 0$

$\Rightarrow (x + 2y)^2 + (- x + 2y + 1)^2 = 0$

$\Rightarrow x + 2y = 0 ; - x + 2y + 1 = 0$

$\Rightarrow x = 1/2 ; y = - 1/4$

$\Rightarrow x - 2y = 1$

Q.2) If $\sqrt{5x + 9} + \sqrt{5x - 9} = 3(2 + \sqrt{2})$, then $\sqrt{10x + 9}$ is equal to

A. $3\sqrt{7}$

B. $4\sqrt{5}$

C. $3\sqrt{31}$

D. $2\sqrt{7}$

Solution:

$(\sqrt{5x+9}+\sqrt{5x-9})^2 = [3(2+\sqrt{2})]^2$

$\Rightarrow 5x+9+5x-9+2\sqrt{25x^2-81} = 9(6+4\sqrt{2})$

$\Rightarrow 10x+2\sqrt{25x^2-81} = 54+36\sqrt{2}$

Let's try matching first term by equating 10x = 54 and see if it works

$\Rightarrow x = 54/10 = 27/5$

For 2nd term,

$\Rightarrow 2\sqrt{25(27/5)^2-81} = 2\sqrt{27^2-81} = 2*9*\sqrt{9-1} = 18*2*\sqrt{2} = 36\sqrt{2}$

Since x = 27/5 satisfies the equation, $\sqrt{10x+9} = 3\sqrt{7}$

Q.3) Let n be the least positive integer such that 168 is a factor of 1134^n. If m is the least positive integer such that 1134^n is a factor of 168^m, then $m + n$ equals

A. 15

B. 12

C. 24

D. 9

Solution:

$168 = 2^3*3*7$

$\Rightarrow 1134 = 2*3^4*7$

If 168 is a factor of 1134^n, then minimum value of n = 3 to match power of 2

If 1134^n is a factor of 168^m, then $2^n * 3^{4n}*7^n$ must divide $2^{3m}*3^m*7^m$ completely

That means, $n \le 3m,\ 4n \le m,\ n \le m$

constraint equation, $4n \le m$

If n =3 (minimum value) , then minimum value of m = 12

⇒ m + n = 15

Q.4) If x and y are positive real numbers such that $log_x(x^2 + 12) = 4$ and $3log_y x = 1$, then (x+y) equals

A. 11

B. 20

C. 10

D. 68

Solution:

$log_x(x^2 + 12) = 4 \quad \Rightarrow x^4 = x^2 + 12$

$\Rightarrow x^4 - x^2 - 12 = 0$

$\Rightarrow x^4 - 4x^2 + 3x^2 + 12 = 0 \Rightarrow x^2 = 4$ as negative value of x^2 is rejected

$\Rightarrow x = 2$ as x can't be negative in $log_y x$, so $x = -2$ is rejected

$3log_y x = 1$

$\Rightarrow y = x^3 = 8$

$\Rightarrow y + x = 10$

Q.5) The number of integer solutions of equation $2|x|(x^2 + 1) = 5x^2$ is

Solution:

$2|x|(x^2 + 1) = 5x^2$

Let's take 3 cases, $x > 0$, $x = 0$, $x < 0$

Case 1: $x > 0$

$\Rightarrow 2x(x^2 + 1) = 5x^2$

$\Rightarrow 2(x^2 + 1) = 5x$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow x = 1/2,\ x = 2$

Case 2: $x < 0$

$\Rightarrow \ -2x(x^2+1) = 5x^2$

$\Rightarrow 2(x^2+1) + 5x = 0$

$\Rightarrow 2x^2 + 4x + x + 2 = 0$

$\Rightarrow x = -1/2, x = -2$

No solutions for negative values of x

Case 3: $x = 0$

0 value of x satisfies the equation

3 integer solutions ⇒ 0 , 2, -2

Q.6) The equation $x^3 + (2r+1)x^2 + (4r-1)x + 2 = 0$ has -2 as one of the roots. If the other two roots are real, then the minimum possible non-negative integer value of *r* is

Solution:

$x^3 + (2r+1)x^2 + (4r-1)x + 2 = 0$

Manipulating the polynomial a little so that we can factorize it with x+2 (-2 is a solution)

$\Rightarrow x^3 + x^2 + 2rx^2 + 4rx - x + 2$

$\Rightarrow x^3 + 2x^2 - x^2 + 2rx^2 + 4rx - 2x + x + 2$

$\Rightarrow x^2(x+2) - x^2 - 2x + 2rx^2 + 4rx + x + 2$

$\Rightarrow x^2(x+2) - x(x+2) + 2rx\ (x+2) + x + 2$

$\Rightarrow [x^2 + x(2r-1) + 1](x+2)$

If other 2 roots are real, then $x^2 + x(2r-1) + 1$ will have real roots

$D = (2r-1)^2 - 4 > 0$

$\Rightarrow 4r^2 - 4r - 3 > 0$

$\Rightarrow (2r+1)(2r-3) > 0$

$\Rightarrow r < -1/2;$ or $r > 3/2$

Which means minimum possible non negative integer value of r can be 2

Q.7) Let α and β be the two distinct roots of the equation $2x^2 - 6x + k = 0$, such that $\alpha + \beta$ and $\alpha\beta$ are the distinct roots of the equation $x^2 + px + p = 0$. Then, the value of $8(k - p)$ is

Solution:

$2x^2 - 6x + k = 0$

$\alpha + \beta = - b/a = 3$

$\alpha\beta = c/a = k/2$

Now using these values in the next equation as sum and product of the roots:

$(\alpha + \beta) + (\alpha\beta) = - p \quad \Rightarrow 3 + k/2 = - p$

$(\alpha + \beta) * (\alpha\beta) = p \quad \Rightarrow 3k/2 = p$

Solve for the answer

Q.8) In an examination, the average marks of 4 girls and 6 boys is 24 . Each of the girls has the same marks while each of the boys has the same marks. If the marks of any girl is at most double the marks of any boy, but not less than the marks of any boy, then the number of possible distinct integer values of the total marks of 2 girls and 6 boys is

A. 20

B. 22

C. 21

D. 19

Solution:

Given $(4g + 6b)/10 = 24; \qquad b \le g \le 2b$

Now we have to find integer values for $2g + 6b = 4g + 6b - 2g = 24 - 2g$

Solving for constraint values

first $g = b \quad \Rightarrow 10g = 240 \quad \Rightarrow g = 24 \qquad \Rightarrow 2g = 48$

now $g = 2b \quad \Rightarrow 7g = 240 \quad \Rightarrow g = 240/7 \qquad \Rightarrow 2g = 480/7 \simeq 68.6$

Now for $24 - 2g$ to vary, let’s see how 2g varies

maximum integer value of 2g is 68 and minimum integer value of 2g is 48

Total 21 values

Q.9) The minor angle between the hour hand and minute hand of a clock was observed at 8:48 am. The minimum duration, in minutes, after 8.48 am when this angle increases by 50% is

A. 36/11

B. 4

C. 24/11

D. 2

Solution:

Angle made by minute hand wrt noon position: $48/60 \ * \ 360^{0} \ = \ 288^{0}$

Angle made by hour hand wrt noon position: $8/12 \ * \ 360^{0} + \ 48/60 \ * \ 30^{0} = \ 264$

After ‘t’ minutes, angle will change to:

$$288^{0} + \ t/60 \ * \ 360^{0} \ - \ 264^{0} - \ t/60 \ * \ 30^{0} \ = \ 1.5 \ * \ (288 \ - \ 264)^{0}$$

$$11t/2 \ = \ 0.5 \ * \ 24$$

Q.10) The salaries of three friends Sita, Gita and Mita are initially in the ratio 5:6:7, respectively. In the first year, they get salary hikes of 20%, 25% and 20%, respectively. In the second year, Sita and Mita get salary hikes of 40% and 25%, respectively, and the salary of Gita becomes equal to the mean salary of the three friends. The salary hiked of Gita in the second year is

A. 28%

B. 26%

C. 30%

D. 25%

Solution:

Let’s say the salaries are 5x, 6x and 7x.

Salaries after first year hike becomes: 6x, 7.5x and 8.4x

Salaries after second year hike becomes: 8.4x, y and 10.5x (y is unknown which is Gita’s salary)

y = (8.4x + 10.5x + y)/3

y = 9.45x, which is 26% higher from 7.5x

Q.11) A mixture P is formed by removing a certain amount of coffee from a coffee jar and replacing the same amount with cocoa powder. The same amount is again removed from mixture P and replaced with same amount of cocoa powder to form a new mixture Q. If the ratio of coffee and cocoa in the mixture Q is 16 : 9 , then the ratio of cocoa in mixture P to that in mixture Q is

A. 5 : 9

B. 1 : 2

C. 4 : 9

D. 1 : 3

Solution:

Assume first time x amount of coffee was removed from 1 unit of total mixture, x<1

In mixture P, coffee : cocoa = $(1 - x) : x$

In mixture Q,

coffee = $(1 - x) - x * (1 - x) = 1 - 2x + x^2$

and cocoa = $1 - (1 - 2x + x^2) = 2x - x^2$

so coffee : cocoa = $\frac{1-2x+x^2}{2x-x^2} = \frac{16}{9}$

$9 - 18x + 9x^2 = 32x - 16x^2 \quad \Rightarrow 25x^2 - 50x + 9$

$x = 1/5\ since\ x < 1$

cocoa in mixture P to Q = $\frac{x}{2x-x^2} = \frac{1}{2-x} = 5 : 9$

Q.12) Gita sells two objects A and B at the same price such that she makes a profit of 20% on object A and a loss of 10% on object B. If she increases the selling price such that objects A and B are still sold at an equal price and a profit of 10% is made on object *B*, then the profit made on object *A* will be nearest to

A. 47%

B. 49%

C. 42%

D. 45%

Solution:

Assume SP for both = 100

CP for A = 250/3

CP for B = 110

New SP for B = 121

New SP for A = SP for B = 121

Profit % on A = $\frac{121-\frac{250}{3}}{\frac{250}{3}}$ * 100 = 45.2%

Q.13) Brishti went on an 8-hour trip in a car. Before the trip, the car had travelled a total of x km till then, where *x* is a whole number and is palindromic, i.e., remains unchanged when its digits are reversed. At the end of the trip, the car had travelled a total of 26862 km till then, this number again being palindromic. If Brishti never drove at more than 110 km / h , then the greatest possible average speed at which she drove during the trip, in km / h , was

A. 90

B. 100

C. 80

D. 110

Solution:

Maximum distance she would have covered = 8*110 = 880

Minimum value of 'x' = 26862 - 880 = 25988

First digit (left to right) of x has to be 2, that means the unit digit is 2 as well.

Second digit could be 5 or 6. To minimize x, let's assume it is 5.

So x could be of structure 25p52 but it is not possible since minimum value of x is 25988

So taking second possibility of digit 6, x would be of structure 26p62

Minimum value of p could be 0

Distance travelled = 26862 - 26062 = 800

Speed = 800/8 = 110

Q.14) The amount of job that Amal, Sunil and Kamal can individually do in a day, are in harmonic progression. Kamal takes twice as much time as Amal to do the same amount of job. If Amal and Sunil work for 4 days and 9 days, respectively, Kamal needs to work for 16 days to finish the remaining job. Then the number of days Sunil will take to finish the job working alone, is

Solution:

Let's say they will take a, s and k no of days to finish the job

k = 2a

since their rate of work are in HP, $s = (k + a)/2$

$s = 3a/2$

$\frac{4}{a} + \frac{9*2}{3a} + \frac{16}{2a} = 1$

$\frac{24+36+48}{6a} = 1$

$a = 18;\ s = 27$

Q.15) Arvind travels from town A to town B, and Surbhi from town B to town A, both starting at the same time along the same route. After meeting each other, Arvind takes 6 hours to reach town B while Surbhi takes 24 hours to reach town A. If Arvind travelled at a speed of 54 km/h, then the distance, in km, between town A and town B is

Solution: Let;s say the distance is x km

Let's say they meet after time 't' and Surbhi's speed is 's'

$54t + st = 54(t + 6) = s(t + 24) = x$

$s = x/(t + 24)$

$t(54 + \frac{x}{t+24}) = x \Rightarrow t(54 + \frac{54(t+6)}{t+24}) = 54(t + 6)$

$t^2 + 24t + t^2 + 6t = t^2 + 30t + 144$

$t^2 = 144;\ t = 12$

$x = 54 * (12 + 6) = 972$

Q.16) Anil invests Rs. 22000 for 6 years in a certain scheme with 4% interest per annum, compounded half-yearly. Sunil invests in the same scheme for 5 years, and then reinvests the entire amount received at the end of 5 years for one year at 10% simple interest. If the amounts received by both at the end of 6 years are same, then the initial investment made by Sunil, in rupees, is

Solution:

$22000 * \left(1 + \frac{4}{200}\right)^{12} = P\left(1 + \frac{4}{200}\right)^{10}\left(1 + \frac{10}{100}\right)$

$22000 * (1.02)^2 = P * (1.1)$

P = 20808

Q.17) Let C be the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ and L be the locus point of intersection of a pair of tangents to C with the angle between the two tangents equal to 60^0 . Then, the point at which L touches line x=6 is

A. (6, 6)

B. (6, 8)

C. (6, 4)

D. (6, 3)

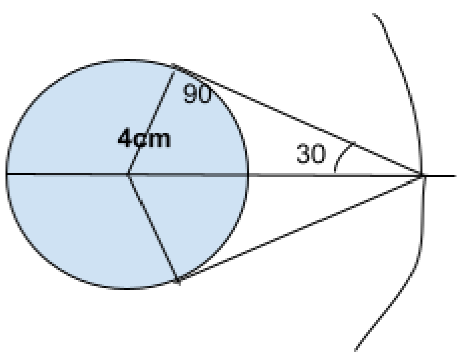
Solution:

Rearranging the equation to identify center and the radius length

$(x - (-2))^2 + (y - 3)^2 = 16$

So the center of the circle is (-2,3) and radius is 4 units.

Now the described locus is also going to be a circle with the same center but at a bigger radius and radius can be calculated easily using basic trigonometry



Radius of the bigger circle = $4/sin(30)$ = 8

So equation of locus is: $(x - (-2))^2 + (y - 3)^2 = 8^2$

Now substituting x = 6, we get y = 3

Q.18) A quadrilateral ABCD is inscribed in a circle such that AB:CD=2:1 and BC:AD=5:4. If AC and BD intersect at the point E, then AE:CE equals

A. 2:1

B. 5:8

C. 8:5

D 1:2

Solution:

You can use the property: Angles subtended by same chord are equal giving:

∠ABE = ∠DCE

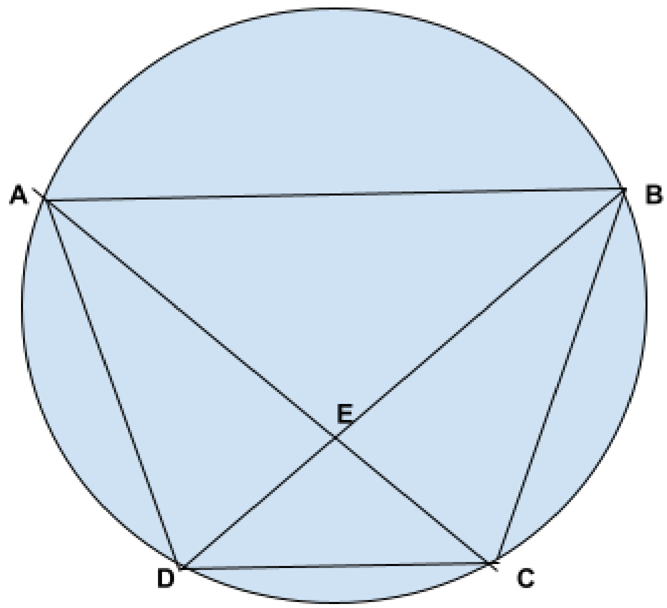
∠BAE = ∠CDE

∠DBC = ∠CAD

∠ACB = ∠BDA

Also opposite angles are equal giving ∠AEB = ∠DEC, ∠AED = ∠BEC

AEB and DEC are similar triangles, similarly AED and BEC are similar triangles.



$$\frac{AB}{DC} = \frac{2}{1} = \frac{AE}{DE} = \frac{BE}{CE}$$

$$\frac{BC}{AD} = \frac{5}{4} = \frac{BE}{AE} = \frac{CE}{DE}$$

$$\frac{AE}{DE} * \frac{DE}{CE} = \frac{2}{1} * \frac{4}{5} = \frac{8}{5}$$

Q.19) In a right-angled triangle ABC, the altitude AB is 5cm, and the base BC is 12cm. P and Q are two points on BC such that the areas of triangle ABP, ABQ and ABC are in arithmetic progression. If the area of triangle ABC is 1.5 times the area of triangle ABP, the length of PQ, in cm, is

Solution:

Since the altitude is not going to change for all triangles ABP, ABQ & ABC (it will be the same perpendicular Ab), and their areas are in AP. That means BP, BQ & BC are in arithmetic progression (using area = bh/2, if h is same then b must be in AP to keep areas in AP)

if area of triangle ABC = 1.5* area of triangle ABP ⇒ BC = 1.5*BP and AQ must be average of two which is 1.25*BP

Also BC = 12cm

PQ = BQ-BP = 0.25*BP =BC/6 = 12/6 = 2

Q.20) The number of all natural numbers up to 1000 with non-repeating digits is

A. 648

B. 585

C. 504

D. 738

Solution:

let's take all single digit ones first = 9

two digits = 9*9 (fill any digit except 0 on tens place and remaining digits along with 0 at unit's place)

three digits = 9*9*8

= 9 + 9*9 + 9*9*8 = 738

Q.21) For some positive and distinct real numbers x,y and z, if $\frac{1}{\sqrt{y}+\sqrt{z}}$ is the arithmetic mean of $\frac{1}{\sqrt{x}+\sqrt{z}}$ and $\frac{1}{\sqrt{y}+\sqrt{x}}$, then the relationship which will always hold true, is

A. y,x and z are in arithmetic progression

B. $\sqrt{x}$, $\sqrt{y}$ and $\sqrt{z}$ are in arithmetic progression

C. x,y and z are in arithmetic progression

D. $\sqrt{x}$, $\sqrt{z}$ and $\sqrt{y}$ are in arithmetic progression

Solution:

$$\frac{2}{\sqrt{y}+\sqrt{z}} = \frac{1}{\sqrt{x}+\sqrt{z}} + \frac{1}{\sqrt{y}+\sqrt{x}}$$

Multiply the equation with $(\sqrt{y} + \sqrt{x})(\sqrt{z} + \sqrt{x})(\sqrt{y} + \sqrt{z})$on both the sides

$$2(\sqrt{y} + \sqrt{x})(\sqrt{z} + \sqrt{x}) = (\sqrt{y} + \sqrt{x})(\sqrt{z} + \sqrt{y}) + (\sqrt{y} + \sqrt{z})(\sqrt{z} + \sqrt{x})$$

$$2x + 2\sqrt{xz} + 2\sqrt{xy} + 2\sqrt{yz} = y + \sqrt{xy} + \sqrt{xz} + \sqrt{yz} + z + \sqrt{zx} + \sqrt{zy} + \sqrt{yx}$$

$$2x = y + z$$

Q.22) A lab experiment measures the number of organisms at 8 am everyday. Starting with 2 organisms on the first day, the number of organisms on any day is equal to 3 more than twice the number of the previous day. If the number of organisms on the n^{th} day exceeds one million, then the lowest possible value of n is

Solution:

$T(n) = 2 * T(n-1) + 3$

So first day $T(1) = 2$

$T(2) = 2 * T(1) + 3 = 2 * 2 + 3 = 7$

$T(3) = 2 * T(2) + 3 = 2 * (2 * 2 + 3) + 3 = 17$

$T(4) = 2 * T(3) + 3 = 2 * (2 * 2 * 2 + 2 * 3 + 3) + 3 = 37$

$T(n) = 2 * T(n-1) + 3 = 2^{n+1} + 2^{n-1} - 3$ (figure out a pattern hat fits in terms of n, it will take trial and error and you need to check with more values of n so that pattern satisfies all values)

Now check with different values of n, if T(n) > 10,00,000

$2^{20} = 2^{10} * 2^{10} = 1024 * 1024 > 1Mn$

$2^{19} < 1Mn$

$T(19) = 2^{20} + 2^{18} - 3 > 1Mn$

$T(18) = 2^{19} + 2^{17} - 3 < 1Mn$

Answer Keys

Q.No.	Quant
1	A
2	A
3	A
4	C
5	3
6	2
7	6
8	C
9	C
10	B
11	A
12	A
13	B
14	27
15	972
16	20808
17	D
18	C
19	2
20	D
21	A
22	19